Social Data Management
Network Robustness

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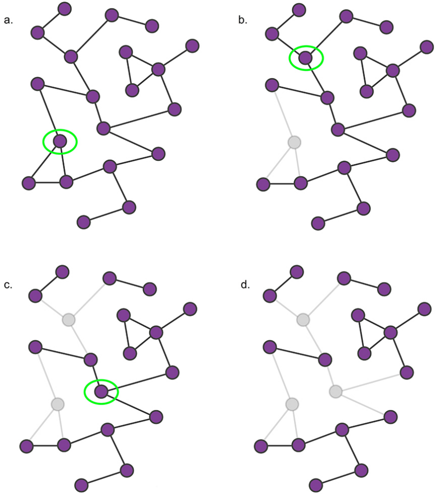
Robustness

Robustness is a central issue in network science.

What happens to a network if some parts of it are *removed*?

- mutations in medicine
- network attack in online social networks
- diseases, famines, wars, ...

Robustness



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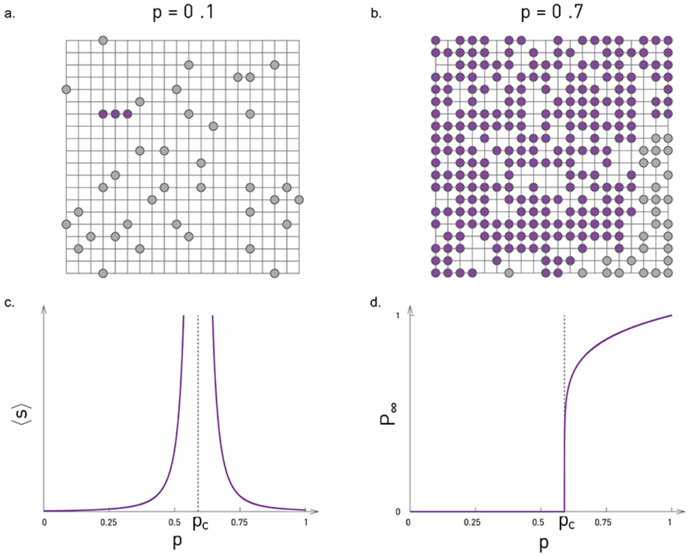
Percolation: term coming from statistical physics, applied in our case: what is the *expected size of the largest cluster* and *the average cluster size*

Example: a square lattice, where "pebbles" are places with probability p at random intersections. If two or more pebbles are connected they form clusters. As p approaches a critical value p_c, a large cluster emerges.

Percolation in Lattices



Percolation in Lattices

We track:

- largest cluster size $\langle s \rangle \sim |p - p_c|^{-\gamma_p}$ – diverges as we approach p_c
- oder parameter $p_\infty \sim (p - p_c)^{\beta_p}$ – probability that a pebble belongs to the larges cluster
- correlation length $\xi \sim |p - p_c|^{-\nu}$ – mean distance between two pebbles belonging to the same cluster

γ_p, β_p, and ν are critical exponents – they characterize the behavior near the critical point

Percolation theory says that the exponents are universal: independent of p_c or the nature of the lattice.

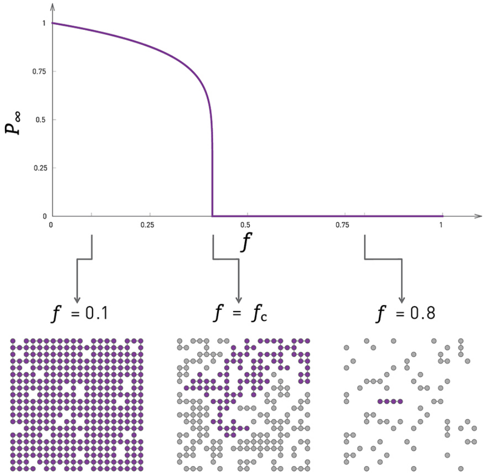
Percolation and Robustness

Inverse percolation: what happens when we remove a fraction f of nodes from the giant component of the lattice

As f increases, the lattice is more and more likely to break up in tiny components

However, the process is not gradual! It is characterized by a critical threshold f_c at which point the lattice is broken.

Inverse Percolation in Lattices



$0 < f < f_c$:

There is a giant component.

$P_\infty \sim |f - f_c|^\beta$

$f = f_c$:

The giant component vanishes.

$f > f_c$:

The lattice breaks into many tiny components.

Random networks under random node failures have the same exponents as the infinite-dimensional percolation.

The critical exponents in random networks are $\gamma_p = 1$, $\beta_p = 1$ and $\nu = 1/2$.

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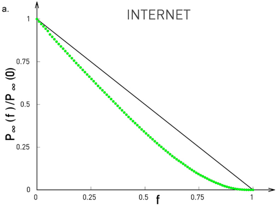
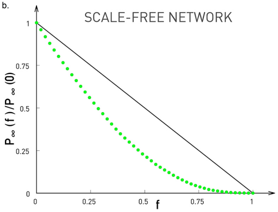
Scale-Free Network and Random Removals

What happens to scale-free networks under random removals? Empirical results show that they are surprisingly resilient. Why?





Molloy-Reed Criterion

f_c in scale free networks is extremely high.

Molloy-Reed criterion: a randomly wired network has a giant component if:

$$\kappa = \frac{\langle k^2 \rangle}{k} > 2; \tag{1}$$

this works for any degree distribution p_k.

For a random network:

$$\kappa = \frac{\langle k \rangle (1 + \langle k \rangle)}{\langle k \rangle} = 1 + \langle k \rangle > 2,$$

or

$$\langle k \rangle > 1.$$

Applying Molloy-Reed in Random Networks

We can apply the criterion to a network with arbitrary degree we have that:

$$f_c = 1 - \frac{1}{\kappa - 1}; \tag{2}$$

depending only on k and k^2.

In a random network:

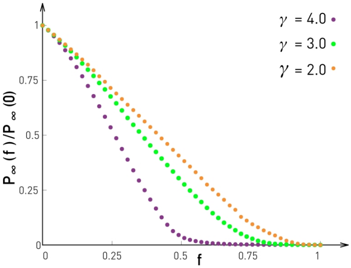
$$f_c = 1 - \frac{1}{\langle k \rangle}.$$

We only need to remove a finite number of nodes, and f_c is higher as the network is denser

Applying Molloy-Reed in Scale-Free Networks

In scale-free networks, f_c depends on the degree exponent γ:

$$f_c = \begin{cases} 1 - \frac{1}{\frac{\gamma-2}{3-\gamma} k_{\text{min}}^{\gamma-2} k_{\text{max}}^{3-\gamma} - 1} & 2 < \gamma < 3 \\ 1 - \frac{1}{\frac{\gamma-2}{\gamma-3} - 1} & \gamma > 3 \end{cases}$$



For $\gamma > 3$, $f_c \to \infty$, meaning that we have to remove almost all nodes in order that the network breaks.

Main takeaway: scale-free networks are resilient under random removals, we can remove an arbitrary number of nodes.

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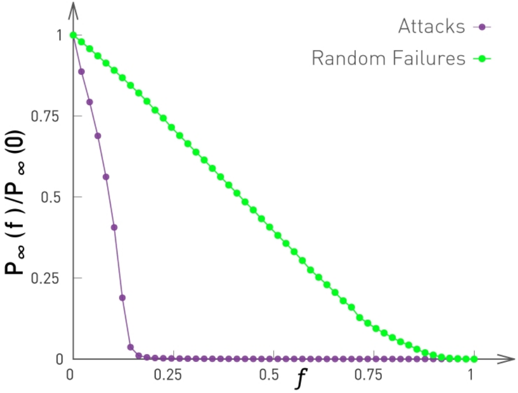
f_c under Attacks

What happens when we attack the network (we choose deliberately the nodes, prioritizing *high degree nodes*?

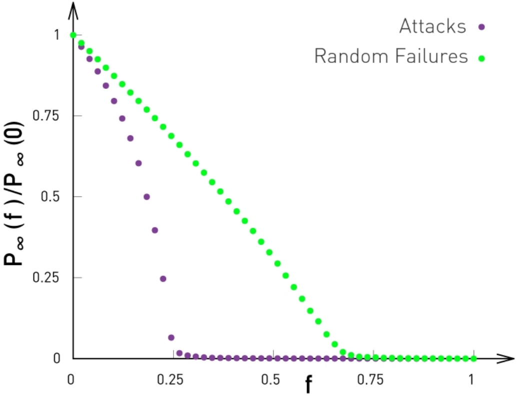
How does f_c change?

Network	**Random**(pred.)	**Random**(real)	**Attack**
Internet	0.84	0.92	0.16
Power Grid	0.63	0.61	0.20
Email	0.69	0.92	0.04
Protein	0.66	0.88	0.06

Attacks: Scale-Free Networks



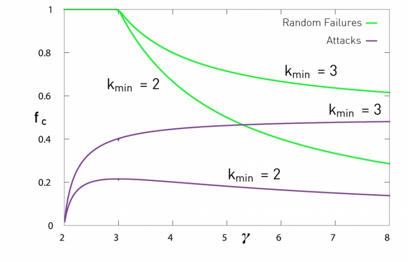
Attacks: Random Networks



Critical Threshold Under Attack

Using the fact that, for large γ the scale-free networks resemble random networks, so random failures and targeted attacks are indistinguishable when $\gamma \to \infty$:

$$f_c \to 1 - \frac{1}{k_{\text{min}} - 1}. \tag{3}$$



Cascading Failures

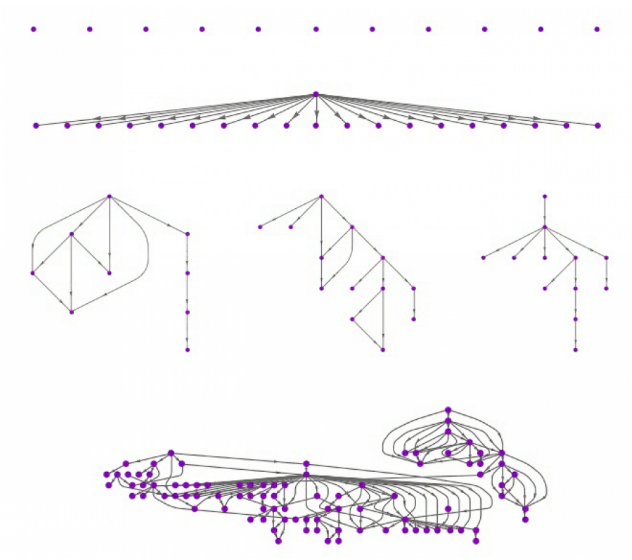
Once an attack is perpetrated, some failures are cascading: the neighbours of the attacked node can fail, which triggers cascades on their neighbours etc.

Examples of cascading failures:

- blackouts on power grids
- denial of service attacks
- information cascades in social networks, viruses
- financial crises

Common characteristic: all the cascading failure follow power laws.

Information Cascades



Acknowledgments

Figures in slides 4, 7, 10, 13, 16, 20, 21, 22, and 24 taken from the book "Network Science" by A.-L. Barabási. The contents is partly inspired by the flow of Chapter 8 of the same book.

`http://barabasi.com/networksciencebook/`

References i

Bakshy, E., Hofman, J. M., Mason, W. A., and Watts, D. J. (2011).
Everyone's an influencer: Quantifying influence on twitter.
In *Proceedings of the ACM International Conference on Web Search and Data Mining*, pages 65–74.

Bollobás, B. and Riordan, O. (2003).
Robustness and vulnerability of scale-free random graphs.
Internet Math., 1(1):1–35.

Callaway, D. S., Newman, M., Strogatz, S. H., and Watts, D. J. (2000).
Network robustness and fragility: Percolation on random graphs.
85(25):5468–5471.

Molloy, M. and Reed, B. (1995).
A critical point for random graphs with a given degree sequence.
Random Struct. Algorithms, 6(2-3):161–180.