



Algorithms for Data Science

Web Advertising

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Adwords

Banner Ads

First iteration: **banner ads** (around 1995)

The screenshot shows the homepage of the French newspaper Le Monde. At the top, the masthead "Le Monde" is centered. To its left is a small icon and the text "Consultez le journal". To its right are links for "Se connecter" and "S'abonner". Below the masthead is a navigation bar with categories: ACTUALITÉS, ÉCONOMIE, VIDÉOS, OPINIONS, CULTURE, M LE MAG, SERVICES, and a search icon. A row of article teasers follows, each with a date, a thumbnail, and a headline. The main content area is dominated by a large blue banner for "amazon prime day 13 et 14 oct.". The banner features the Amazon logo, the text "Deux jours de Ventes Flash exceptionnelles sur les produits que vous aimez", and several article teasers. On the left side of the banner, there's a section for "amazon prime day 13 et 14 octobre" with a "Voir les ventes flash" button. Below this, there's a section for "La faillite de la démocratie aux États-Unis, épisode 2 : la disparité des règles de vote selon les États" with a "Voir les ventes flash" button. In the center, there's a section for "Des chiffres trompeurs montrant la « probabilité de survie » des personnes infectées par le Covid-19" with a "Voir les ventes flash" button. On the right side, there's a section for "Notre voix, un instrument si fragile" with a "Voir les ventes flash" button. At the bottom of the banner, there's a section for "L'UE adopte des critères mon" with a "Newsletter Le Brief du Monde" button. The banner also features illustrations of a Prime Day delivery truck and a house.

Le Monde

Consultez le journal

Se connecter S'abonner

ACTUALITÉS ÉCONOMIE VIDÉOS OPINIONS CULTURE M LE MAG SERVICES

13 oct. 12 militaires russes tués

13 oct. Le Japon, modèle de respect de la forme ?

13 oct. **Alerte** Serie : Les faillites de la démocratie américaine, épisode 2 : la disparité des règles de vote risque de entraîner la confusion

13 oct. Le FBI perçoit une rupture « longue et difficile »

14 oct. Grippe : vaccins, aller-vous vous faire vacciner ?

Store >

amazon prime day 13 et 14 oct.

Deux jours de Ventes Flash exceptionnelles sur les produits que vous aimez

amazon prime day 13 et 14 octobre

Voir les ventes flash

Exclusivement pour les membres Amazon Prime

La faillite de la démocratie aux États-Unis, épisode 2 : la disparité des règles de vote selon les États

Voir les ventes flash

Exclusivement pour les membres Amazon Prime

Des chiffres trompeurs montrant la « probabilité de survie » des personnes infectées par le Covid-19

Voir les ventes flash

Exclusivement pour les membres Amazon Prime

Notre voix, un instrument si fragile

Voir les ventes flash

Exclusivement pour les membres Amazon Prime

10 ans de lecture

L'UE adopte des critères mon

Newsletter Le Brief du Monde


Banner Ads

First iteration: **banner ads** (around 1995)

- charging per 1,000 “impressions” (**clicks**)
- **CPM** – cost per thousand impressions (as in TV, print media)
- **untargeted** vs. **demographically targeted**
- low **click through rates** – low return on investment

Performance-Based Advertising

Second iteration: **ads on search results** (around 2001)



[All](#) [Images](#) [Shopping](#) [Maps](#) [News](#) [More](#) [Settings](#) [Tools](#)

About 37,100,000 results (0.45 seconds)

Ad · www.go-sport.com/running/chaussures

Chaussures de Running - GO Sport : Magasins de Sport

Chaussures de Running - Retrouvez la Sélection de Produits sur GO Sport. Articles & Matériel de Sport : Randonnée, le Fitness, Tennis, Running, Sports collectifs. Paiement sécurisé. Retrait en magasin. Marketplace. Livraison en 24h. Paiement en 3X sans frais.

★★★★★ Rating for go-sport.com: 4.5 · Order accuracy: 95–100%

📍 Les Ulis · 11 locations nearby

Chaussures Running

Commandez en ligne les Meilleures Marque de Chaussures de Running

Opération Automne

Profitez de Réduction sur une Sélection de Produits

Vélo, VTT, VTC

Découvrez notre gamme de vélos en stock et livrés

Rentré

Les Ind... une bor

www.go-sport.com · [running](#) · [chau...](#) · [Translate this page](#)

Chaussures Running - achat pas cher - GO Sport

Chaussures Running toujours au meilleur prix sur Go-Sport.com – 24h/24 bénéficiez d'un large choix d'articles de sport – Commande en ligne et Livraison ...

Homme · Chaussures Homme · Femme · Running REVOLUTION 5

www.decathlon.fr · Tous les sports · Running route

Chaussures et baskets running | DECATHLON

Chaussures et baskets running. Pour courir, rien de plus important qu'une bonne paire de chaussures de course à pied. Vous trouverez ici notre large gamme de ...

★★★★★ Rating: 4.4 · 18,636 reviews

Chaussures de running Homme · Chaussures de Running Femme










www.i-run.fr · [chaussures_homme](#) · [Translate this page](#)

Chaussures de sport : running, randonnée, fitness pour homme

Reebok Floatride Run : Idéales pour les hommes ayant une foulée universelle, ces chaussures sont toutes indiquées pour les séances de course à pied sur routes ...

Ads · See chaussures run...

[Products](#) [Comparison Sites](#)

 <p>NIKE Chaussures de running hom...</p> <p>€43.99</p> <p>Intersport</p> <p>By Google</p>	 <p>Nike Baskets Revolution 5...</p> <p>€54.90</p> <p>La Redoute</p> <p>★★★★★ (1k+)</p> <p>By Keyade</p>	 <p>ASICS Gel - Cumulus 21 Gs...</p> <p>€49.00</p> <p>outlet.asics.com/...</p> <p>★★★★★ (18)</p> <p>By Google</p>
 <p>Chaussures Running &...</p> <p>€30.00</p> <p>Decathlon.fr</p> <p>By Yteo</p>	<p>SALE</p>  <p>Nike Nike Air Zoom Pegasus ...</p> <p>€59.50 €85</p> <p>Sarenza</p> <p>Free shipping</p> <p>By Feed Price</p>	<p>SALE</p>  <p>Asics Gel Pulse 11 M Jaune/or E...</p> <p>€65.00 €100</p> <p>i-Run.fr</p> <p>★★★★★ (141)</p> <p>By Productcaster</p>
<p>SALE</p>  <p>Reebok Floatride Run</p>	<p>SALE</p>  <p>Adidas Ultraboost</p>	 <p>New Balance Fresh Foam</p>

Performance-Based Advertising

Second iteration: **ads on search results** (around 2001)

- advertisers **bid** on **search keywords**
- on **click** – highest bidder ad is shown
- charging only if add is clicked
- adopted by Google around 2002 – **Adwords**

Performance-Based Advertising

Part of Web 2.0 – huge industry (several billion \$)

Problem: what ads to show for a given query

- another related problem: which search terms should an advertiser bid on, and for how much
- part of computational game theory

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Data Streams: limited resources to process data as it comes

Online algorithms

- decision must be made **immediately** as data comes
- vs. **offline** – data is processed in its entirety

Greedy Algorithm for Online Optimization Problems

Optimization problem: maximizing or minimizing an **objective function** on the data

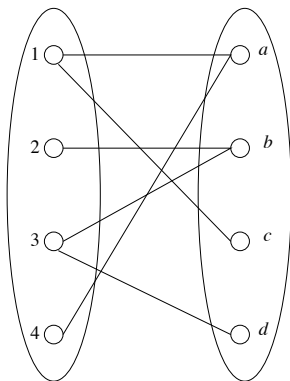
Greedy algorithm: take decision locally, by **optimizing** only based on the **current** element and the past

Not always optimal vs. offline algorithm:

- **competitive ratio:** the ratio between the offline solution and the online solution **over all inputs** $c = \max_G \frac{|M_g|}{|M_o|}$

Matching Problem

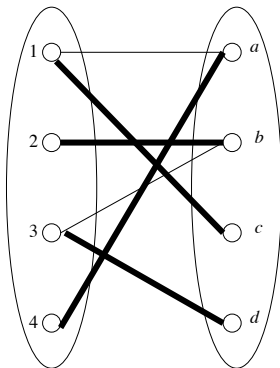
Bipartite Graph: a graph $G(V_1 \cup V_2, E)$ having two disjoint sets of nodes V_1 and V_2 and edges **only** havins one endpoint in V_1 and one in V_2 , i.e., $E \subseteq V_1 \times V_2$



Matching Problem

Matching: choosing a **subset of the edges** in the bipartite graph s.t. **no node has more than two edges** in the matching

- **perfect** – every node is in the matching
- **maximal** – has the largest number of edges possible



Greedy Algorithm for Matching

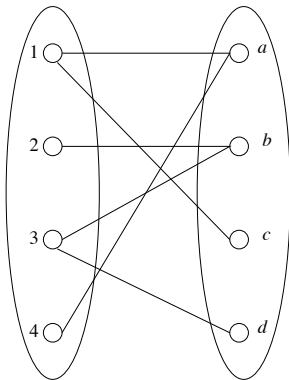
Offline case: algorithms for finding maximal matchings are $\mathcal{O}(n^2)$, where $n = |E|$

Online case: can use the **greedy** algorithm:

1. consider the edges in the order they arrive
2. add edge (x, y) only if neither x nor y are endpoints

Example of Greedy Matching

Edges arrive in the order: $(1, a)$, $(1, c)$, $(2, b)$, $(3, b)$, $(3, d)$, $(4, a)$



Result of greedy matching: $(1, a)$, $(2, b)$, $(3, d)$ – **not maximal**

Competitive Ratio of Greedy Matching

M_o – maximal matching, M_g – greedy matching

L – left nodes matched in M_o but not in M_g

R – right nodes connected to any node in L

Claim: every node in R is matched in M_g

- **prove by contradiction:** assume it is not the case
- then there will exist edge (l, r) , $l \in L$
- then, it should be matched (neither is added to the matching)
- contradiction!

Competitive Ratio of Greedy Matching

Claim: every node in R is matched in M_g

- $|M_o| \leq |M_g| + |L|$ – only nodes in L can be matched in M_o
- $|L| \leq |R|$ – in M_o , all nodes in L are matched
- $|R| \leq |M_g|$ – every node in R is matched in M_g

– this gives us $|M_g| \geq \frac{|M_o|}{2}$ – **lower bound** on the competitive ratio

But $1/2$ is also an **upper bound** – can find a counter example

Competitive ratio is then exactly $1/2$

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Adwords Problem

Problem: match queries in a search engines with advertisers

We have:

- a set of **bids** by advertisers for search queries
- **click-through rate** for each advertiser-query pair
- **budget** for each advertiser (time, money, etc.)
- **limit on the number of ads** to be displayed

Adwords Problem

Problem: match queries in a search engines with advertisers

Restrictions on the set of advertisers:

- **the size is under the limit** of number of ads
- each advertiser in the set **has bid on the query**
- each advertiser **has enough budget left over**

Adwords Setting

1. **stream of queries** arrives at search engines q_1, q_2, \dots
2. advertisers **bid** on each query
3. when q_i arrives search engine picks a subset of advertisers

Objective: maximize search engine revenue

If we consider queries as being the “left” side and advertisers the “right” side in a bipartite graph – **online bipartite matching**

- **weighted case:** the matching depends on the CTR and the budget

Adwords in Practice

In practice: combine CTR and bid – **expected revenue**

- **value** of an ad – expected revenue
- **revenue** to the search engine – sum of values of matched ads

Advertiser	CTR	Bid	CTR \times Bid
A	0.02	7.5	0.15
B	0.05	5.0	0.25
C	0.01	1.0	0.01

Measuring CTR

Value of an ad is directly linked to the CTR rate

- high bid is useless if the CTR is very low

Click-through rate is measured historically – **difficult problem**

- **explore**: do we try an ad to measure the CTR rate for future campaigns?
- **exploit**: do we use the current known CTR rate, even if they could be outdated?

Greedy Algorithm

Setting:

- there is one ad shown for each query
- all advertisers have the same budget B
- all ads have same CTR
- value is then the same

Greedy algorithm:

- pick any advertiser who has bid for that query
- same **competitive ratio** as in online matching – $1/2$

Worst-case Greedy

Advertiser A: bids on query x , budget 4 **Advertiser B:** bids on queries x and y , budget 4

Stream: $x\ x\ x\ x\ y\ y\ y\ y$

Greedy choice:

- worst case: $B\ B\ B\ B\ \dots$
- **optimal:** $A\ A\ A\ A\ B\ B\ B\ B$
- **competitive ratio:** $1/2$

BALANCE algorithm [Mehta et al., 2007]

BALANCE Algorithm:

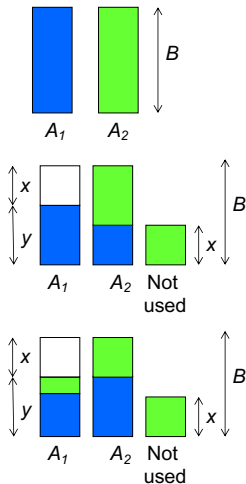
- assign query to the bidder having the most budget left
- **competitive ratio** $3/4$
- **tie breaking**: must be deterministic

Previous example:

- if A is preferred to B : $A B A B B B \dots$
- establishes an **upper bound** on competitive ratio for 2 bidders

BALANCE – Lower Bound for 2 Bidders

Assumption: advertisers A_1, A_2 budget B (consumed by the optimal algo), revenue $2B$



BALANCE must **exhaust the budget of at least one bidder**, e.g., A_2

Case of assigned bids ($x + y = B$):

- **at least half of the queries are assigned to A_1 :** $y \geq B/2$, so $y \geq x$
- **more than half of the queries are assigned to A_2 :** remaining budget of A_2 is less than $B/2$, so $x \leq B/2$, so $y \geq x$

Minimal BALANCE revenue at $x = y = B/2$,
revenue $3B/2$ **competitive ratio** $\frac{3B/2}{2B} = 3/4$

BALANCE – Multiple Bidders

In the general case, BALANCE competitive ratio is not much lower than the simple case:

- competitive ratio: $1 - 1/e = 0.63\dots$
- no online algorithm has a better competitive ratio

BALANCE – Worst Case for Multiple Bidders

Advertisers: $N - A_1, \dots, A_N$, each having budget $B > N$

Queries: N rounds of B queries

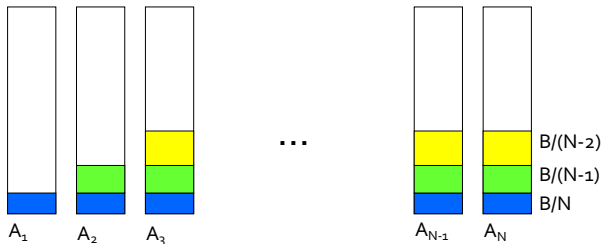
Bids: round i - bidders A_i, \dots, A_n

Optimum allocation: allocate round i queries to A_i

- **revenue** $N \cdot B$

BALANCE – Worst Case for Multiple Bidders

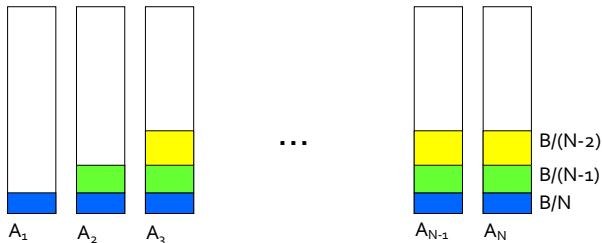
BALANCE allocation



- BALANCE assigns queries in round k to $N - (k - 1)$ advertisers
- sum of allocation to each advertiser A_k, \dots, A_N : $S_k = \sum_{i=1}^{k-1} \frac{B}{N-(i-1)}$
- the smallest k at which $S_k \geq B$ is the point after which no advertisers can be allocated $k = N(1 - 1/e)$

BALANCE – Worst Case for Multiple Bidders

BALANCE allocation



- after $k = N(1 - 1/e)$ we cannot get any revenue
- **total revenue:** $B \cdot N(1 - 1/e)$
- upper bound on **competitive ratio:** $1 - 1/e$

BALANCE with Arbitrary Bids

BALANCE works well when bids are 0 or 1

- if arbitrary bids, it can fail and have competitive ratio 0

Example:

- advertisers A_1, A_2 , one query q arriving 10 times
- A_1 : bids 1, budget 110
- A_2 : bids 10, budget 100
- **optimal**: assign all queries to A_2 , revenue 100
- **BALANCE**: assigns all queries to A_1 , revenue 10

Generalized BALANCE

BALANCE can be generalized to arbitrary bids:

- bid x_i , budget b_i , amount spend so far m_i
- **fraction of leftover budget** $f_i = 1 - m_i/b_i$
- for a query q , use $\psi_i(q) = x_i(1 - e^{-f_i})$

Decision:

- allocate query q to bidder i having largest value of $\psi_i(q)$

Same competitive ratio: $1 - 1/e$

Adwords Implementation

In practice

- advertisers **bid of sets of words**
- if a search query contains exactly those words – the advertiser becomes a bidder
- can use **distributed hash tables**
- queries can be distributed on **several machines** also – multiple streams

Another applications:

- Google also matches **ads to emails** – much harder problem (mails are much larger)

Acknowledgments

The contents follows Chapter 8 of [Leskovec et al., 2020]. Figures in slides 11, 12, 14, 26, 29, and 30 are taken from <https://www.mmds.org/>



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